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Planetary Ball Transmissions: Strength Calculations

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Abstract—The design of transmissions with intermediate rollers is considered. A structure for spherical planetary ball transmission is proposed. Formulas for the forces on the rollers are proposed, along with a method of calculating the strength of transmission components and an algorithm for determining the engagement coefficient.

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Mechanical transmissions with rollers have several benefits: they are compact, multiflux systems with a high engagement coefficient. In such transmissions, the slipping friction in the couplings is replaced by rolling friction, with corresponding reduction in the frictional losses. Modular design of the multistage reducing elements is possible because the input and output shafts are coaxial. Moreover, interest in such transmissions is associated with the comprehensive study of traditional involute gear transmissions, the search for new solutions, and the explosive development of technology. In the past, the manufacture of complex profiles required special attachments, but today high-precision numerically controlled machine tools permit the production of surfaces of any complexity.

Many patents have been obtained for roller transmissions. Most studies focus on the theory of mechanisms and machines, including analysis of their structure and kinematics but generally not their dynamics. As a rule, this reduces to determining the relation between forces by the kinetostatic method. Strength calculations for the components of roller transmissions may be encountered in some dissertations and monographs, but this topic requires more attention.

In the present work, we consider the structure and operating principle of planetary ball transmissions. In particular, we consider transmissions in which the rollers do not simply replace the gear teeth but form a system of bodies that are not physically coupled but move consistently, so as to simulate the motion of a flexible element in a wave transmission. Such transmissions may be regarded as a distinct group. However, they have evident analogies with other well-known mechanisms and transmissions. They may be regarded as cam mechanisms. Likewise, gear transmissions may be regarded as a multiply repeated cam mechanism, according to [1]. The forward and inverse cam mechanisms are connected in series, as is clear in considering separately the interaction of each individual roller with the basic components. Such transmissions may also be represented as sets of wedge mechanisms, where the roller (slider) is in contact with several inclined surfaces. However, systematic analysis of the whole mechanism reveals an analogy with a wave transmission. We have already characterized it as a planetary transmission, since the kinematic dependence complies completely with the Willis formula and hence the torques are interdependent. The model of the transmission takes the form of two periodic curves that are closed on a cylindrical surface. These curves are the generatrices of the tracks on the cams over which the balls move. In that case, the number of periods of the curves corresponds to the number of teeth, while the dimensions of the rollers (the satellites) have no influence on the kinematic parameters of the transmissions.

Transmissions with radial and axial roller motion are widely used [2]. There has been less study of spherical transmissions, but their use improves mechanisms such as hinges of equal angular velocity, compensating clutches, and mechanisms for harvesting motion in eccentric transmissions with an additional reduction stage.

The structure of a spherical planetary ball transmission is shown in Fig. 1. Rotation of input shaft *1* turns rigidly attached eccentric *2* and internal cam *3*. Rollers *4* move in the annular slot of the internal cam. They come into contact with the end working surfaces of external cam *5*, rigidly attached to housing *6*. Under the action of the applied links, the rollers also move along the slots of shaft *7*, thereby forcing it to turn, with speed reduction. The slots in shaft *7* are produced by means of a spherical mill and are distributed over the internal cylindrical surface with a uniform angular spacing. Disk *8* connects slotted shaft *7* to output shaft *9*. The gear ratio of this system is $i = 1 + z_3$, where z_3 is the number of projections (periods) of the end surface of external cam *5*.

The inclined slot on the cylindrical surface takes the form of a single-period sinusoid; the interaction

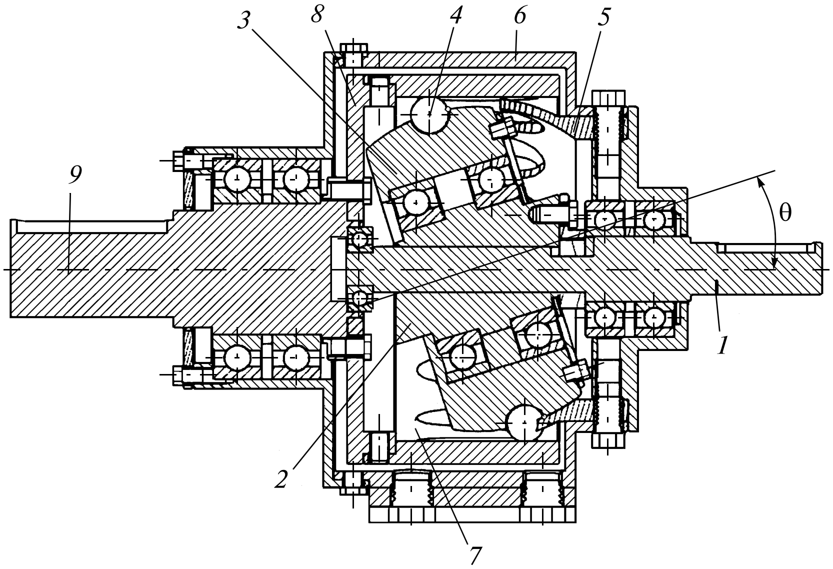

Fig. 1. Gear system with spherical planetary ball transmission.

curve is a multiperiodic sinusoid. The equation describing the spherical surface of the multiperiodic interaction curve with the annular slot (circle) is complex.

The performance of this transmission is assessed on the basis of the contact strength; the flexural strength of the projections on the external cam; and the wear resistance of the working surfaces of the cams and the shaft with the slots.

Algorithms for determining the basic geometric parameters of cylindrical planetary ball transmissions may be found in [3]. Check calculations of the transmission's strength are based on preliminary force analysis. We consider the interaction of ball *4* with three basic components of the transmission (Fig. 2).

According to the frictional model of the transmission, we obtain formulas for the forces on the rollers from the shaft as a result of the slots (N_2), the internal cam (N_1), and the external cam (N_3)

$$N_2 = \frac{M_2}{R_2 k \cos\beta}; \tag{1}$$

$$N_1 = \frac{N_2\left(f\sin\beta - f - \dfrac{\cos\beta}{\sin\alpha_{3m}}(\cos\alpha_{3m} + f\sin\alpha_{3m})\right)}{f\sin\alpha_{1m} - \cos\alpha_{1m} - \dfrac{(\sin\alpha_{1m} + f\cos\alpha_{1m})(\cos\alpha_{3m} + f\sin\alpha_{3m})}{\sin\alpha_{3m}}}; \tag{2}$$

$$N_3 = \frac{N_2\left(-\dfrac{\cos\beta(f\sin\alpha_{1m} - \cos\alpha_{1m})}{\sin\alpha_{1m} + f\cos\alpha_{1m}} + f\sin\beta - f\right)}{\cos\alpha_{3m} + f\sin\alpha_{3m} - \dfrac{\sin\alpha_{3m}}{\sin\alpha_{1m} + f\cos\alpha_{1m}}(f\sin\alpha_{1m} - \cos\alpha_{1m})}, \tag{3}$$

where M_2 is the torque at the output shaft; k is the number of rollers transferring the load (the engagement coefficient); f is the complex frictional coefficient (which is the same for all the pairs), including both slipping and rolling friction; α_{1m} is the mean inclination of the generatrix of the tracks of the internal cam; α_{3m} is the mean inclination of the working end surfaces of the external cam.

The mean inclinations are calculated as $\alpha_{im} = \arctan(2z_iA_i/(\pi R_i))$, where $i = 1$ for the internal cam and 3 for the external cam; A is the amplitude of the curve; R is the radius of the generatrix of the cylindrical surface where the centers of the rollers are located; z is the number of periods of the curves.

In the model, we make the following assumptions: (1) $\beta = \arcsin(1 - r_{z2}/r_s)$ is the angle between force

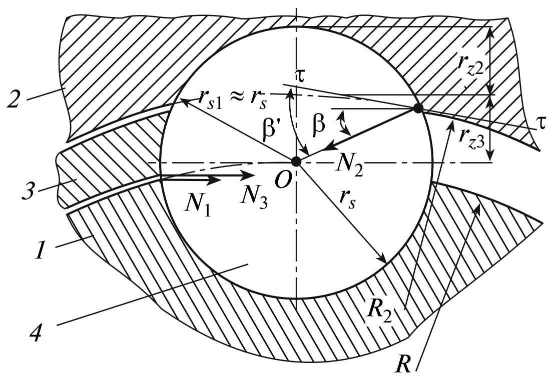

Fig. 2. Interaction of rollers with basic components of transmission.

vector N_2 and the horizontal, rather than the tangent to a circle of radius R_2, since the difference between β and β' is slight; (2) forces N_1 and N_3 are assumed to be horizontal.

Analysis of Eqs. (1)–(3) shows that N_2 is greatest: it exceeds N_3 by a factor of 1.3–2 for different gear ratios (2–100). The maximum stress in the contact zone for steel parts may be estimated by means of the Hertz formula after appropriate modification for planetary ball transmissions [4]

$$\sigma_{H\max} = 188.715(\nu_1\nu_2)^{-1}N_2^{1/3}(2r_s^{-1} - r_{s1}^{-1})^{2/3}, \quad (4)$$

where ν_1 and ν_2 depend on the geometry of the contacting bodies and may be found as a function of r_s/r_{s1} from Table 1 in [4]. Here r_s is the ball radius; r_{s1} is the channel radius for the slotted shaft.

Then, on the basis of the contact strength in Eq. (4), the minimum permissible ball diameter d_s takes the form

$$d_s = 4\left([\sigma_{co}]^{3/2}\left(\frac{188.715}{\nu_1\nu_2}\right)^{-3/2}N_2^{-1/2} + r_{s1}^{-1}\right)^{-1}, \quad (5)$$

where $[\sigma_{co}]$ is the permissible contact stress.

For planetary ball transmissions, the permissible contact stress is determined as for ball transmissions with the same operating principle [6]: $[\sigma_{co}] = k_{sc}[\sigma_{co60}]$, where $[\sigma_{co60}]$ is the permissible contact stress when the hardness of the contacting surfaces is at least 60 *HRC*; k_{sc} is the factor by which the permissible contact stress is reduced when the surface hardness is more than 60 *HRC*. The permissible contact stress is 2500–3000 MPa in prolonged operation and 4000 MPa in short-term operation. The factor k_{sc} varies from 1 (at 60–62 *HRC*) to 0.415 (at 35 *HRC*) and is determined from the softest part in the transmission. Finally, the ball diameter must be refined and rounded to the standard value in accordance with the required range. For ball–screw transmissions, to avoid premature ball and channel wear, the guideline $N = 2660d^2$ is recommended, where N is the normal force and d is the ball diameter [5]. The results obtained from Eq. (5) when $[\sigma_{co}] \approx 3000$ MPa are the same as the results when $r_s/r_{s1} = 0.99$.

We must also check how the flexural strength is affected by the projections on the external cam that form its periodic profile. We consider the projection as a cantilever beam, analogous to a gear tooth. We neglect the curvature of the projections in the plane perpendicular to the axis of the transmission. Attention focuses on the cross section at the base of the projection. The maximum bending force on the projection from the roller arises when the ball passes the median engagement line. The inclination α_3 is a maximum here: $\alpha_{3\max} = \arctan(z_iA_i/R_i)$. The flexural stress σ_F is determined on the tensile side of the projection

$$\sigma_F = N_3\left(\frac{6A\sin(\alpha_{3\max})}{0.25d_s\left(\frac{2\pi R}{z_3} - d_s\right)^2} - \frac{\cos(\alpha_{3\max})}{0.25d_s\left(\frac{2\pi R}{z_3} - d_s\right)}\right). \quad (6)$$

In Eq. (6), we take account of the geometric relations $r_{z2} = r_{z3} = 0.5r_s$ characterizing the increasing depth of the rollers in the channel of the slotted shaft and the contact length of the ball with the working surfaces of the external cam. The determination of the permissible flexural stress for planetary ball transmissions resembles the method for gear transmissions in State Standard GOST 21354–75.

The wear resistance of the transmission components is determined from the condition $N_i v_{sl_i} \le [Nv_{sl}]$, where N_i is the force on the roller from the basic components of the planetary ball transmissions; v_{sl} is the slip velocity at contact [2]; $[Nv_{sl}]$ characterizes the permissible wear resistance of the working surfaces of the cams and slotted shaft.

It is important to determine the engagement coefficient, which characterizes the number of rollers simultaneously engaged with the three basic elements of the planetary ball transmission. The total number of balls in a single section of the transmission is $b = z_1 + z_3 = 1 + z_3$. However, not all the rollers participate in the load in the transmission. We have developed the following algorithm to determine the engagement coefficient k.

First, consider the maximum projection height z_H of the cam (Fig. 3), measured from the median line—that is, the circle in the plane xOy with its center at point O and with radius R equal to the radius of the generatrix of the cylindrical surface where interaction curves with z_1 and z_3 periods are located. The projection is formed after a tool (spherical or cylindrical mill) with a radius equal to the roller radius r_s passes through the central multiperiodic curve. The initial data are as follows: the equation of the projection onto the plane of the multiperiodic curve $z = f(x)$; and the parameters A, z_3, r_s, R. The height z_H is determined by

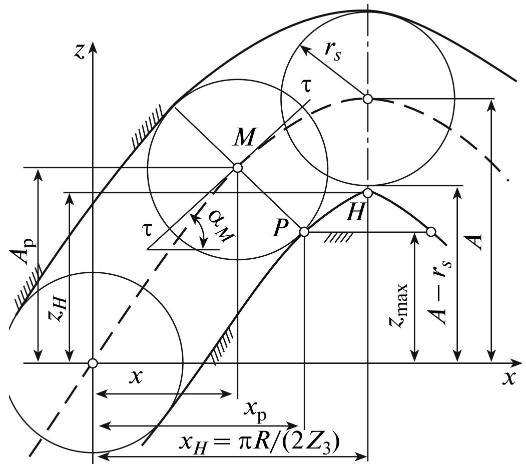

Fig. 3. Calculation of engagement coefficient k.

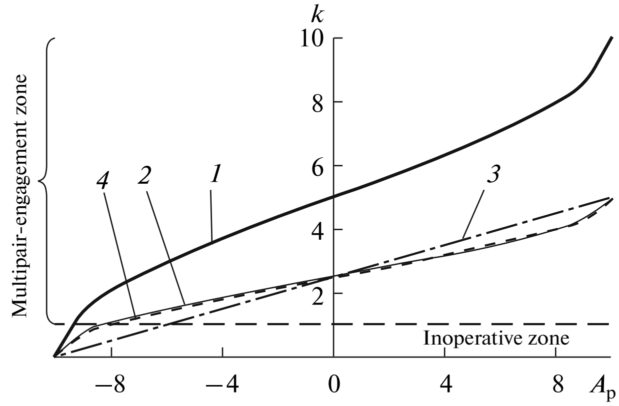

Fig. 4. Variation in engagement coefficient for a spherical planetary ball transmission with different interaction curves: (*1*) sinusoid with $z_3 = 9$; (*2*) sinusoid with $z_3 = 4$; (*3*) piecewise helical curve with $z_3 = 4$; (*4*) curve of the transmission in Fig. 1, with $z_3 = 4$.

numerical solution of a system of equations with three unknowns x, α, and z_H

$$\begin{cases} \tan\alpha = \frac{d}{dx}f(x); \\ x + r_s\sin\alpha = \pi R/(2z_3); \\ z_H = f(x) - r_s\cos\alpha. \end{cases} \quad (7)$$

Here α is the inclination of the curve at the given point—in other words, the angle between the tangent τ–τ to the curve at the given point (for example, point M) and the horizontal axis (Fig. 3).

After determining z_H, the actual profile height is reduced so as to remove the sharpened sections. The maximum profile height is z_{max}. Then we write Eq. (7) in the form

$$\begin{cases} \tan\alpha = \frac{d}{dx}f(x); \\ x + r_s\sin\alpha = x_p; \\ z_{max} = f(x) - r_s\cos\alpha, \end{cases} \quad (8)$$

where the unknowns are x, α, and x_p.

Determining the unknowns, we calculate the maximum height A_p at the projection of the curve onto the plane by substituting x into the equation of the plane projection $z_{max} = f(x)$. The height A_p is found from the median line. This result indicates what part of the curve is active in load transmission. In the operation of the transmissions, balls below level A_p participate in engagement, while higher balls lose contact with the working surfaces, although they continue to move along the track of the internal cam and in the slots of the slotted shaft.

In Fig. 4, we plot k as a function of A_p for transmissions with $A = 10$ mm and $R = 20$ mm, in the case of different interaction curves. The program for determining k simulates consistent stepwise motion of the rollers over different curves. In each step, the number of balls below A_p is determined. The mean number of such balls over the operating cycle (a circle rotation of the driveshaft) is assumed to be the engagement coefficient k.

Analysis shows that the dependence of the engagement coefficient on A_p corresponds to a halfperiod of the initial curve along the vertical axis. To obtain analytical expressions for k, we must express the horizontal component k in terms of the vertical component A_p in the initial equations.

The engagement coefficient takes the following forms:

for a sinusoid

$$k = \frac{b}{\pi}\arcsin\left(\frac{A_p}{A}\right) + \frac{b}{2};$$

for a piecewise helical curve (a combination of inclined segments)

$$k = \frac{b}{A}A_p + \frac{b}{2} = \frac{b(A_p + A)}{2A};$$

for the curve in Fig. 1

$$k = \frac{b}{\pi}\arcsin\left(\sin\left(\frac{A_p}{R}\right)\Big/\sin\left(\frac{A}{R}\right)\right) + \frac{b}{2}.$$

In the latter case, $k = 6.35$ when $z_1 = 1$, $z_3 = 14$, $A = 15$ mm, $R = 50$ mm, $r_s = 6.15$ mm, and $b = 15$.

A 3D model of the system based on a spherical planetary ball transmission is shown in Fig. 5. This system has been manufactured and tested in the laboratory. The tests show that, as for gear transmissions

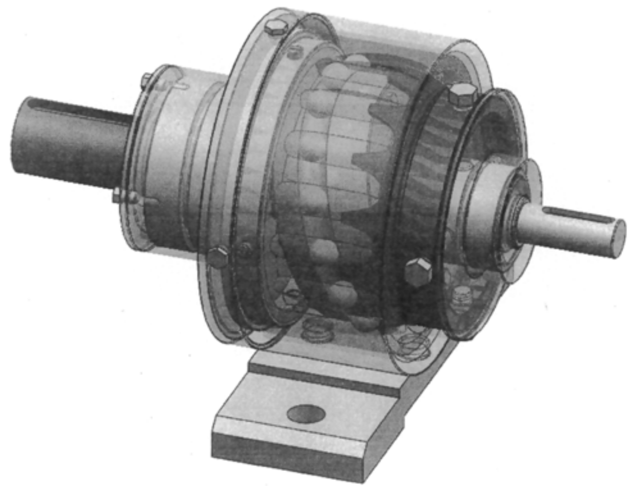

Fig. 5. Gear system with spherical planetary ball transmission.

operating in an oil bath, the contact stress is the determining factor in strength calculations for planetary ball transmissions. For identical torques, the size and mass of planetary ball transmissions are theoretically smaller by a factor of 3–4 than for planetary nonhelical gear transmissions. Experimental comparison with a single-stage three-satellite transmission (i = 15) shows that the difference is not so perceptible: the planetary ball transmission is smaller by a factor of 1.1–1.3. This is associated with the nonuniform load distribution between the rollers. This nonuniformity must be taken into account by means of an empirical coefficient or compensated by high precision of manufacturing and assembly.

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